WORLD METEOROLOGICAL ORGANIZATION

INSTRUMENTS AND OBSERVING METHODS

REPORT No. 19

SOME GENERAL CONSIDERATIONS AND SPECIFIC EXAMPLES IN THE DESIGN OF ALGORITHMS FOR SYNOPTIC AUTOMATIC WEATHER STATIONS

by

D.T. ACHESON

1987

WMO/TD - No. 230

CONTENTS

1.Ø INTRODUCTION

1.1 If asked "What is the weather like outside?", most people might respond "Cloudy, warm, and some what breezy.". In the same situation, a weather observer would reply "2Ø SCT M9Ø OVC 6K l73/75/68/3Øl Ø/ØØ4". Despite the (contrived) fact that both messages contain about the same number of characters, the observer's reply obviously includes much more specific and detailed information. His senses were augmented by a number of sensors which must have included a ceilometer, barometer, thermometer, hygrometer, anemometer, and wind vane in this example. Furthermore, he had been carefully trained to observe visibility and identify obstructions to visibility, to use tables to compute sea level pressure and altimeter setting, and to code and format a message properly. While not explicit, he had also exercised considerable judgement in controlling the quality of data in the message. For example, in this message temperature exceeds dewpoint, wind direction lies in the range of ØØ to 36, sea level and altimeter setting pressures are compatible, and no sensors were recognized as defective which would have resulted in missing message elements.

1.2 This observer is not unique. Thousands of them throughout the world routinely produce frequent observations in a standard format for exchanging high quality information obtained by uniform observing techniques. To accomplish this, very considerable resources have been devoted over very many years to standardize content, quality, and format. As we move toward automated observation of the atmosphere, we are faced with the need to preserve much of what has already been standardized and to devise means of accomplishing the functions of the human observer with modern data processing equipment.

1.3 Not surprisingly, much of the existing technology and standardized manual techniques can be utilized by automatic weather stations. These include many sensors, standard computations for deriving elements of the message, and the message format itself. That is not to say the adaptation for automation is always trivial. Not all sensors interface easily with automated equipment. Analytic expressions for computations presently embodied in tables must be recovered or discovered. The rules for coding messages must be expressed in computer languages with degrees of precision, completeness, and unambiguousness not demanded by natural language instructions prepared for human observers.

1.4 On the other hand, automation creates the need to quantify functions performed by humans for which no standards have been established. Among these are data quality control and the extraction of statistics. It is easy enough to teil the observer to report an average temperature and daily maximum and minimum, but much more detail and precision is needed to instruct a computer to accomplish the same. Further more, some human functions cannot be automated using either current or foreseeable technologies. The observation of cloud types is an obvious example.

1.5 The same functions executed in the current mix of sensors, humans, and communications equipment should also be found in automatic weather stations. These functions are:

- transducing atmospheric variables
- conditioning transducer outputs
- converting or displaying transducer outputs
- linearizing transducer outputs
- controlling data quality
- extracting statistics, such as the average, from data
- deriving related variables
- formatting the message
- checking the contents of the message
- transmitting the message

1.6 The order in which these functions are arranged is only approximately sequential. Certainly the first listed above must always be first and the last, last. While linearization may immediately follow or be inherent in the transducer, it should not follow the extraction of an average value. Also, either the conventional mix of men and machines or a fully automated station can operate in a diminished capacity without incorporating some of these functions. For example, either one could operate without data quality control and a check of message content, but only by jeopardizing credibility.

1.7 Recognize that in both the conventional and automatic weather station, these functions may be distributed. Sensors such as a laser ceilometer may incorporate all the functions listed above for presentation to an observer or ingestion by another processor. Alternatively, raw transducer outputs from the full sensor complement of a station may feed directly into a single processor embracing all the remaining functions.

1.8 The following sections elaborate on some of the functions listed above. Message formatting and transmission are not discussed at all. only those features of transducers, signal conditioning, linearization, and conversion which impact upon the principal topics are introduced. The focus is upon data quality control, extraction of statistics, and derivation of related quantities. Data quality control incorporates checking the contents of the message.

1.9 Before proceeding further, the reader should clearly understand the viewpoint from which this document was written. In processing meteorological data, there is usually a correct procedure, algorithm, or approach and an infinity of approximations ranging in validity from extremely good to useless. The correct approach is greatly preferable for a number of reasons:

- Considering the current electronic technology and prices and the steady trend toward greater performance at lower costs, it is difficult to justify choosing an approximation if the correct approach is known. When the complement of sensors for a very modest automatic weather station costs at least $1ØØØØ, its housings and shelters and communications equipment a comparable amount, and its installation that amount again, the expenditure of a few hundred dollars to double the processing capability seems reasonable.

- Considerable experience strongly suggests that the correct approach is usually the most efficient in the long term. It is direct, requires a minimum of qualifications, and once implemented correctly needs no further attention.

- Good approximations are accompanied by sound estimates of their range and conditions of validity. It takes considerable skill and experience and effort to derive these estimates.

Accordingly, the subsequent presentations are largely limited to a single, correct approach to the problem under consideration. However, exact solutions do not exist for all problems addressed in the automation of surface observations. Approximations and conditions for their validity are presented when necessary or illustrative.

2.Ø TRANSDUCERS, SIGNAL CONDITIONING, AND CONVERSION

2.1 Everyone recognizes the necessary role of a transducer to sense an atmospheric variable and to convert it quantitatively into a useful and convenient (usually electric) signal. Most users realize that transducers may have secondary responses to the environment, such as temperature dependent calibrations, and their outputs are subject to a variety of errors, such as drift, noise, etc. However, it is not widely appreciated that atmospheric variables fluctuate rapidly and randomly because of ever-present turbulence and that transducer outputs are not faithful reproductions of atmospheric variables because of imperfect dynamic characteristics such as the limited capability of transducers to respond to rapid changes.

2.2 Almost all transducers follow very slow or low frequency changes in the atmosphere but respond with diminished sensitivity to more rapid or higher frequency fluctuations. It is impractical to describe all types of sensor response in this document since the variety is 50 large. For example, adopting the common (and oversimplified) definition of time constant as the time interval required to show 63 percent of a step-function change, cup and propellor anemometer time "constants" vary inversely with wind speed, hygrometer time "constants" depend on temperature and humidity in very complex and poorly understood ways, and underdamped wind vanes oscillate at a frequency proportional to wind speed with the oscillations decaying in time inversely proportional to wind speed.

2.3 Transducers generally require conditioning to amplify their outputs and/or convert from one output form to another; e.g. from resistance to voltage. The circuitry used to accomplish this may also smooth or low-pass filter the signal. 1f the transducer output requires linearization this function may be incorporated in the signal conditioning circuitry and may in turn introduce further filtering.

2.4 A rigorous understanding of the difference between an atmospheric variable and a signal representing it requires complex and sophisticated analysis beyond the scope of this document. To satisfy its purposes, the recognition and qualitative understanding of two characteristics of transducers and signal conditioning circuitry will suffice. First, smoothing or low-pass filtering of a nonlinear and non-constant quantity will introduce errors into averages of that quantity. Second, signals are bandwidth limited. In other words, there is a cutoff frequency above which no significant signal fluctuations occur because none exist in the atmosphere and/or the transducer or signal conditioning circuitry has removed them.

2.5 The next section discusses the importance of linearization before averaging or executing operations similar to averaging such as low-pass filtering and smoothing. 1f ignored, significant systematic errors may be introduced into averages. Non-linear and concurrently slow-responding sensors introduce these errors. The rotating cup anemometer is a well-documented example. The problem cannot be avoided entirely, but its consequences can be minimized. While the use of sensors which respond more rapidly than the atmosphere fluctuates is the correct solution, such sensors with the additionally required properties of high reliability, ruggedness, acceptable cost, etc. are usually not available. As a practical matter for synoptic observations, any sensor responding to atmospheric wavelengths of a few meters or less is adequate. In instrumental terms, this criteria corresponds to distance constants of 3 or 4 meters or time constants of 3 or 4 seconds at wind speeds of 1 meter per second and Ø.3 or Ø.4 seconds at 1Ø meters per second. If this cannot be satisfied, very careful and often extensive analysis may reveal a correct choice of transducer and associated circuitry. For example, the long thermal time constants of some thermometers introduce no error into averages of their outputs.

2.6 To process signals digitally, an analog-to-digital converter (ADC) is needed to interface analog and digital devices. The question arises of how often an ADC input channel should be sampled. The unqualifiedly correct answer is at a rate at least twice the cutoff frequency of the input signal. A simpler and equivalent rule of thumb usually suffices: the sample interval should not exceed the largest among the time constants of all devices and circuitry preceding the ADC in the channel under consideration. 1f the ADC sample rate is smaller than twice the cutoff frequency, unnecessary increases in the variance of the digitized data and <u>all derived quantities and statistics</u> occur. While these increases may be acceptable in particular cases, in others they may not. Proper sampling always ensures minimum variance

2.7 It seems worthwhile to point out that good design may call for incorporating a low-pass filter with a time constant about equal to the sampling rate immediately ahead of the ADC. It is a precautionary measure to minimize effects of noise, especially 5Ø or 6Ø Hz power main pickup by cables connecting sensors to processors and leakage through power supplies.

3.Ø LINEARIZATION

3.1 Because the sequence of operations "average then linearize" produces different results than "linearize then average" when the signal is not constant throughout the averaging period, it is necessary to average linear data. One deals with three sources of nonlinearity. First, many transducers are inherently nonlinear; i.e. their output is not directly proportional (within an additive constant) to the measured atmospheric variable. A thermistor is a simple example. Second, although a sensor may incorporate linear transducers, the variables measured are not linearly related to the atmospheric variable of interest. For example, the photodetector and shaft angle transducer of a rotating beam ceilometer are linear devices, the ceilometer output signal (backscattered light intensity as a function of angle) is nonlinear in cloud height. Third, care must be taken in choosing the atmospheric variable to average. As will be shown below, extinction coefficient, not visibility nor transmittance, is the proper variable to average to produce estimates of average visibility!

3.2 The next table lists several elements of a synoptic observation which are reported as averages and the corresponding linear atmospheric variables.

Element of Observation	Linear Variable (typical units)
station pressure	pressure (millibars, hectopascals)
air temperature	temperature (degrees Celcius)
dewpoint temperature	absolute humidity (kg water vapor/cubic meter)
wind speed and direction	Cartesian components (meters/second)
cloud base height	height (meter)
visibility	extinction coefficient ($meter^{-1}$)

The remainder of this section discusses averaging to obtain reportable values for visibility, wind speed and direction, and dewpoint temperature. In the discussion, upper case letters denote averaged quantities and lower case letters, "instantaneous" variables.

3.3 Visibility

3.3.1 Since visibility depends upon the human observer and the targets and their backgrounds used as well as properties of the atmosphere, a sensor cannot measure visibility. Instead, sensors measure extinction coefficient or some uniquely related quantity such as transmissivity. One then uses various formulae to compute an estimated visibility, called instrumental visibility. The question at hand is should one average instrumental visibilities or average extinction coefficient and then compute an average instrumental visibility. The following example clearly indicates that the latter is correct.

3.3.2 Consider the visibility field shown in Figure 3.1. The banded structure is assumed to extend over a very large area. The width of the bands are not critical, but it is convenient to visualize them as equal and less than a few hundred meters. Assume that instrumental visibility v and extinction coefficient CE are related by:

(3.1) c_E = 3.Ø/v

Over an interval of time long compared to the period of the visibility pattern passing the sensor or over a distance large compared to the wavelength of the visibility pattern shown in the figure, the average of instrumental visibilities V_X is:

(3.2) V_X = (1ØØ + 1ØØØØ)/2 = 5Ø5Ø meters

while the average instrumental visibility V is:

(3.3) V= 3.Ø/C_E = 3.Ø/[(Ø.Ø3 + Ø.ØØØ3)/2 =198 meters

where C_E = average extinction coefficient

It should be obvious that 198 meters is a much more realistic estimate than 5 kilometers. By using similar arguments, one can show that transmissivity is nonlinear as well.

3.4 Wind

3.4.1 The conventional approach to obtaining wind speed and direction averages for coding an observation consists of measuring and averaging wind speed and direction. An obvious problem arises with computing the direction average because the discontinuity between 359 and Ø degrees is a severe nonlinearity. Engineers have devoted a surprising amount of effort and ingenuity to solving this problem in automatic weather stations. An additional and more subtle difficulty exists which the following analysis will reveal.

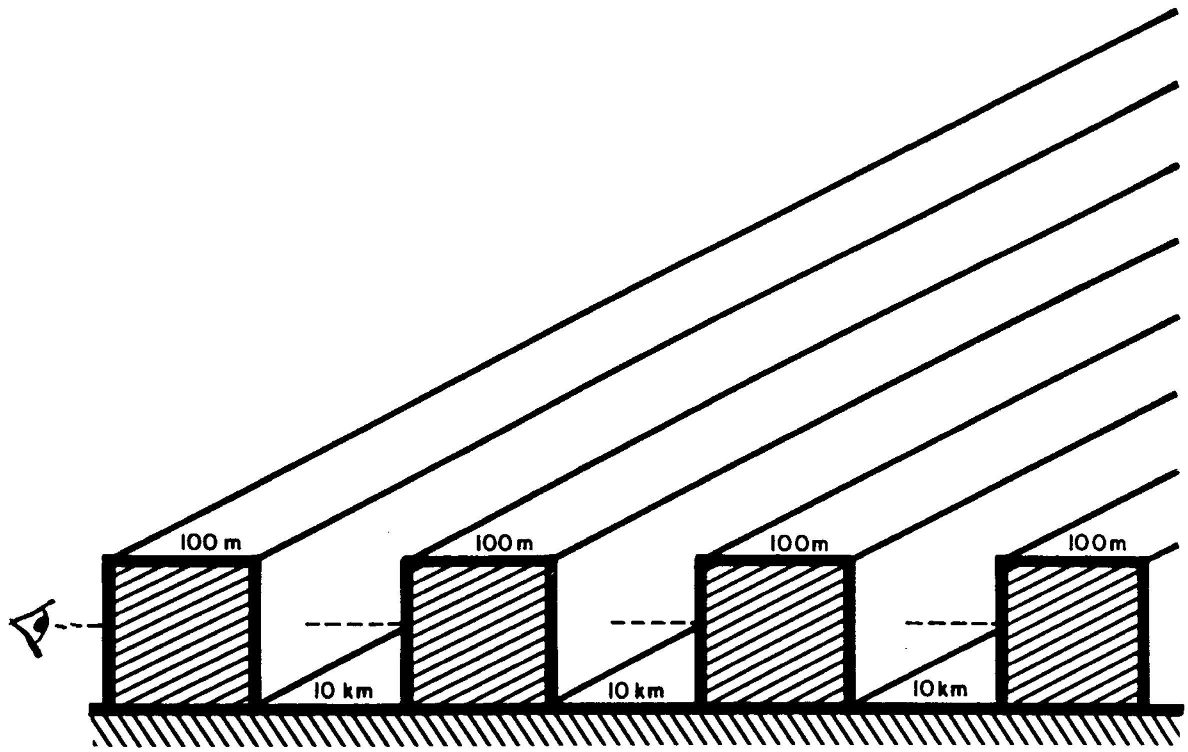


FIGURE 3.1

Visibility field alternating between 100 meters and 10 kilometers

3.4.2 Let u and v represent the Cartesian wind components. The wind speed s is then:

(3.4) $s=(u^2+v^2)^{0.5}$

Writing u=U+u' (sum of average U and fluctuation about the average u') and v=V+v', expanding the square root term in a binomial series, and taking the average of the resulting terms, the average wind speed S becomes:

(3.5) $S=(U^2+V^2)^{.5}[1+.5(V^2/(U^2+V^2))\{d_u{}^2\}+.5(U^2/(U^2+V^2))\{dv^2\}+...]$

where $d_u = u'/(U^2+V^2)^{0.5}$

$d_v = v'/(U^2+V^2)^{0.5}$

{ } = average (alternative notation)

Equation (3.5) expresses the unsatisfactory situation that the average wind speed depends upon the turbulent intensities d_u and d_v. The problem arises because the definition of wind speed in is nonlinear. It is resolved by redefining the average wind speed S as:

(3.6) $S = (U^2+V^2)^{0.5}$

3.5 Dewpoint Temperature

3.5.1 To show that absolute humidity is the correct humidity variable to average, consider a cylinder of moist air of fixed cross-section and length. For simplicity, assume the cylinder to be subdivided into N subvolumes and that the temperature, pressure, and humidity are constant in each, but may vary from one to the next. It is reasonable to require that the average humidity, namely the sum of N humidities divided by N, be identical to the humidity obtained if the moist air from all subvolumes were thoroughly mixed. Absolute humidity satisfies this requirement.

3.5.2 How non-linear are other measures of humidity? To answer this question, a considerable amount of analysis and knowledge of the properties of the atmosphere is needed. Vapor pressure is the easiest case to examine.

3.5.3 For the purposes of this analysis, assume that water vapor behaves as an ideal gas in air. Then the vapor pressure e is related to the absolute humidity d and temperature t by:

(3.7) $e = R_w dt$, where R_w=gas constant for water vapor

Denoting the averages of e, d, and t as E, D, and T respectively and the instantaneous departures of d and t from their average values as d' and t' respectively; equation (3.7) may be rewritten as:

$$(3.8) \quad e = R_w (D+d') (T+t')$$

$$= R_w (DT+Dt' +Td' +d't')$$

Taking the average of both sides of (3.8), using an alternative notation $\{x\}=X$ for an average, and noting that the average of fluctuations about the mean is zero by definition:

$$(3.9) \quad E = R_w DT + R_w\{d't'\}$$

or

$$(3.1\emptyset) \quad E=R_w DT[1+\{(d'/D)(t'/T)\}].$$

The departure from linearity is given by:

$$(3.11) \quad (E-R_w DT)/R_w DT = \{(d'/D)(t'/T)\}$$

The right side of (3.11) is a normalized cross-correlation between humidity and temperature fluctuations. (d'/D) is unlikely to exceed Ø.1 in the rootmean-square sense and (t'/T), Ø.Ø1. Thus, the departure of water vapor pressure from non-linearity is most unlikely to exceed Ø.1 percent, far less than the best attainable sensor accuracies.

3.5.4 In the analyses of wind speed and water vapor pressure nonlinearities, terms involving statistics of turbulent fluctuations of atmospheric variables have occurred. Estimating their magnitude, or limits 0fl their magnitude, accurately will always be difficult. At times, such as in the discussion following equation (3.11), it is easy to see that the term is negligibly small. At other times, such as in equation (3.5), the decision is not dear.

<u>3.6 Choosing the Linear Variable</u>

3.6.1 Since averaging nonlinear variables creates difficulty only when the variable changes during the averaging period, the easiest way to distinguish between linear and nonlinear variables is to examine the consequences of <u>very, even unrealistically, large changes</u> during the averaging period or over a spacial averaging interval or area. This is precisely what was done to demonstrate that extinction coefficient, not instrumental visibility, should be averaged. The same thing easily could have been done for wind speed and direction and for dewpoint temperature. However, the purpose of the more detailed analyses of these two cases was to show that realistic analyses require the expenditure of a considerable effort and a knowledge of the statistical properties of turbulent fluctuations of atmospheric variables. Not only is the requisite knowledge often missing, but even if present one must recognize that the atmosphere has a propensity to exceed our estimates of its behavior. Always averaging linear variables is a far more secure approach.

4.Ø DATA QUALITY CONTROL

4.1 To ensure high quality observations, it is not sufficient to design, test, and correct deficiencies in an automatic weather station prototype before acquiring, installing, and operating a network. During operation, performance deteriorates because of the ageing of components, exposure to untested circumstances, degraded maintenance, and the too frequent failure of production equipment to attain prototype performance levels. One needs the means to monitor the quality of observations continuously during operation.

4.2 The following controls can be implemented systematically:

- Careful prototype design, testing, and correction of deficiencies.
- Thorough and diligent production testing.
- Use of redundant sensors in automatic weather stations, keeping in mind that identical sensors are likely to have similar drifts and biases.
- Incorporating sensor status output signals, in addition to the principal output(s) representing the atmospheric variable, in sensor designs. Sensors often include components other than transducers whose failure seriously degrades or renders useless the principal output. Power supplies, fans, sources of radiant energy, etc. are common examples.
- Identification and labelling of erroneous data prior to further processing. Intra-sensor data editing refers this process applied to the data of a single sensor. Inter-sensor data editing refers to this process applied to data from two or more sensors.
- Processing of edited data to exclude or otherwise properly handle data labelled as erroneous. Intra-sensor data quality control incorporates both this process and editing applied to data from a single sensor. Inter-sensor data quality control is the analagous termininology for two or more sensors.
- Inclusion of processor self-check features.
- Examination, at a central location, of observations from a network of stations to detect sensor drift and bias.
- Establishment and use of good maintenance, repair, and calibration procedures and facilities.

The application of sensor status signals and intra- and inter-sensor data quality control are discussed in greater detail below.

4.3 Sensor Status Signals

4.3.1 As noted above, the principal output of a sensor, the signal representing an atmospheric measurement, often critically depends upon the proper operation of components other than the principal transducer. Power supplies, fans, radiant energy sources, heaters, and coolers are common examples. Inclusion of circuitry or other devices to monitor their status and the routine examination of the status can be a very effective data quality control and maintenance tool. Transmission of the sensor status signal data, either as an appendage to the routine observational message or as a separate clocked or on-request transmission from the automatic stations in a network to central facilities opens a door to novel approaches to the maintenance of meteorological equipment. In addition to that opportunity, the next subsection offers a natural and effective way to use this information in automated data quality control.

4.4 Intra-sensor Data Quality Control

4.4.1 This paper offers no unique, correct approach to intra-sensor data editing. The topic is not well understood and frequently ignored. However, the author is familiar with two measurement systems, both sounding the upper atmosphere, for which editing is absolutely essential - without it the resultant estimates of wind profiles are worthless. In both of these cases, an ever-present background noise alone results in marginal signal-to-noise ratios and marginally accurate wind averages. The frequent interjection of large spurious interference destroys the utility of the data unless effective editing techniques are applied.

4.4.2 While these cases may be extreme and it is difficult to imagine similar problems with signals from many automatic weather station sensors, the author suspects that belief in the validity of this notion often rests more in wishful thinking than fact. Both rotating beam and laser ceilometers operate at marginal signal-to-noise ratios over a significant part of their range during common meteorological conditions and, consequently, are readily victimized by spurious interference. Self-heated lithium chloride and cooled mirror dewpoint hygrometers utilize feedback loops to establish and maintain sensor temperature to ensure equilibrium between sensor and atmospheric water vapor pressures. Yet these feedback loops are characterized by parameters which depend in complex and poorly understood ways upon temperature, pressure, and humidity. Undamped oscillatory behavior, sometimes completely out of control, has been observed. Cognizant readers may be aware of many other examples of sensors producing nonsensical outputs.

4.4.3 Diligent testing and prompt fault correction of prototype and production sensors and their associated circuitry can accomplish a great deal and perhaps excuse the absence of comprehensive intra-sensor data editing. As a minimum, however, the station software should be vigilant, detecting if not correcting serious errors. The procedure outlined in the following subsection will suffice.

4.4.4 Vigilance

4.4.4.1 One seeks an algorithm which will incorporate knowledge of the validity of data, such as would follow from sensor status signals, and produce a good estimates of the quality of the elements of an observation, especially the averages which comprise most of a message. The following will accomplish this.

4.4.4.2 Define the average X of a set of data $x_1, x_2, \ldots, x_N$ as:

$$(4.1)\quad X = \sum_{n=1}^{N} w_n x_n / \sum_{n=1}^{N} w_n$$

where the weights w_n are measures of goodness of the data x_n. One can also define the estimated uncertainty of the average var(X) as:

$$(4.2)\quad \mathrm{var}(X) = 1/\sum_{n=1}^{N} w_n$$

For the simple case of $w_n = 1/\mathrm{var}(x)$ for all n,

$$(4.3)\quad X = 1/\sum_{n=1}^{N} x_n/N \qquad \mathrm{var}(X) = \mathrm{var}(x)/N$$

4.4.4.3 The interesting and useful case occurs by letting $w_n = i_n/\mathrm{var}(x)$, where $i_n=1$ for each acceptable data point and each unacceptable point. Refer to i_n as the "binary worth" of datum x_n. If there are M unacceptable data points, then (4.1) and (4.2) reduce to:

$$(4.4)\quad X = \sum_{n=1}^{N} i_n x_n/(N-M) \qquad \mathrm{var}(X) = \mathrm{var}(x)/(N-M)$$

$$\text{where } \mathrm{var}(x) = \sum_{n=1}^{N} i_n [x_n - X]^2/(N-M-1)$$

Note that var(x) computes using the data without requiring any additional information.

4.4.4.4 One can imagine the following sequence of actions comprising intra-sensor data quality control:

- initialize $i_n = 1$ for all n
- set n = Ø if either:
  - a sensor status signal indicated failure when x_n was observed, or
  - x n is otherwise identified as being in error
- compute the average and the variance of the data var(x)
- compute the variance of the average, var(X)
- use the average if its variance is smaller than or equal to a predetermined accuracy or reject it if larger

How does one otherwise identify x_n as being in error?

4.4.5 Simple Editing

4.4.5.1 Some very simple checks 0n each data sample can detect large errors. Range checks establish if the sample falls outside established limits or differ excessively from a predicted value. Rate-of-change checks compare the sample with the previous valid sample or other value to establish if a plausible rate of change has occurred or not.

4.4.5.2 Establishing limits for range checks is an easy matter if the limits are coarse enough. For example, setting limits at Ø and 2ØØ meters per second for wind speed samples seems valid for any surface observation, but reducing the upper limit to a significantly lower value, say 5Ø meters per second, seems questionable. Yet a single sample in error by 5Ø meters per second among 9 valid ones results in a 5 meter per second error in the wind speed, although if the approach outlined in 4.4.4.4 were in use the estimated variance of the average wind speed, $(5.27\ m/s)^2$, would indicate something is awry. In other words, only the grossest errors are detectable.

4.4.5.3 Range checks for a sample differing excessively from a predicted value and rate-of-change checks are similar in that both presume limits on how quickly an atmospheric variable can change. One obvious difficulty is determining actually how quickly an atmospheric variable can change. This question can be resolved if one knows the response characteristics of the sensor in question, since, in reality, one samples the sensor output, not the atmosphere. Other and more subtle problems await the innocent. The techniques being discussed depend upon comparison with a valid quantity which may be a previous sample or some combination of previous samples such as an average. 1f the quantity is based upon a previous sample or samples containing undetected errors, it is possible to reject all subsequent valid samples. 1f one uses a window of acceptance which widens each time a new sample is identified as erroneous, then the probability of accepting an erroneous sample increases along with the probability of accepting a valid one.

4.4.5.4 These simple editing techniques can be effective when only large, isolated errors occur. They tend to be sensor specific and not amenable to significant refinement or tightening of limits or windows of acceptance. A better and more general approach, made at the expense of conceptual and computational sophistication, is presented in the following subsection.

4.4.6 More Elaborate Editing

4.4.6.1 One can show that the procedure outlined in 4.4.4.4 is equivalent to least squares fitting a zeroth order polynomial (a constant) to the time series. A logical extension of this notion is to extend the fitting to higher order polynomials and so refine the estimated variance of the average. The remaining discussion of this subsection offers a means of accomplishing this task and simultaneously extends the capability to identify outliers and erroneous data.

4.4.6.2 The algorithm proposed below is predicated upon the fact that time series of atmospheric variables exhibit a considerable amount of point-to-point consistency, or temporal continuity, if sampled at a high enough rate. A rate consistent with the considerations of paragraph 2.6 is minimally sufficient. For example, a series of temperature measurements taken each second for several minutes may show a trend and may exhibit observational noise of a few tenths of a degree. But any datum differing from its neighbors by several degrees is obviously suspect and should be disregarded in computing the average. Figure 4.1 illustrates the situation.

4.4.6.3 Figure 4.1 also shows the least squares fitted straight line, the average value, and the residual variance of all the data and the same information with the 3 outliers removed. Obviously, removing the 3 erroneous data points produces much more acceptable results. Exactly what are the least squares fitted straight line and the residual variance?

4.4.6.4 If the data x_n have been observed at a sequence of times t_n, values i_n (Ø or 1) assigned, and a straight line $z_n = a_0+a_1t_n$ drawn (n=1,2,...,N), then the residual variance of the data about the line is given by:

$$(4.5)\quad rvar(x,z) = \sum_{n=1}^{N} i_n\,(x_n-z_n)^2\,/(N-M-2)$$

$$\text{where } N-M = \sum_{n=1}^{N} i_n$$

The least squares line is that one defined by the choice of coefficients $a_Ø$ and a_1 which minimize the residual variance. A variety of well-established procedures exist for finding the coefficients. The concepts of residual variance and least squares fitted curves are readily extended to higher order polynomials.

4.4.6.5 One can easily understand the means of identifying and assigning a binary worth Ø to those points for which sensor status signals indicate failure. The means for identifying and assigning binary worth Ø to outliers is not so obvious. The following algorithm will accomplish the desired result.

(1) Compute the initial values of the least squares coefficients and residual variance for the set of points x_n, i_n and t_n. i_n= 1 unless set to Ø by a sensor status signal. Set n=1.

(2) If i_n = Ø, skip directly to step (6).

(3) Compute the residual of x_n, its departure from the fitted line, $r_n = x_n-z_n$.

(4) If $r_n^2/rvar(x,z)$ is less than or equal to a predetermined threshold, then skip directly to (6).

(5) Set i_n = Ø and recompute the coefficients and residual variance as if x_n never existed.

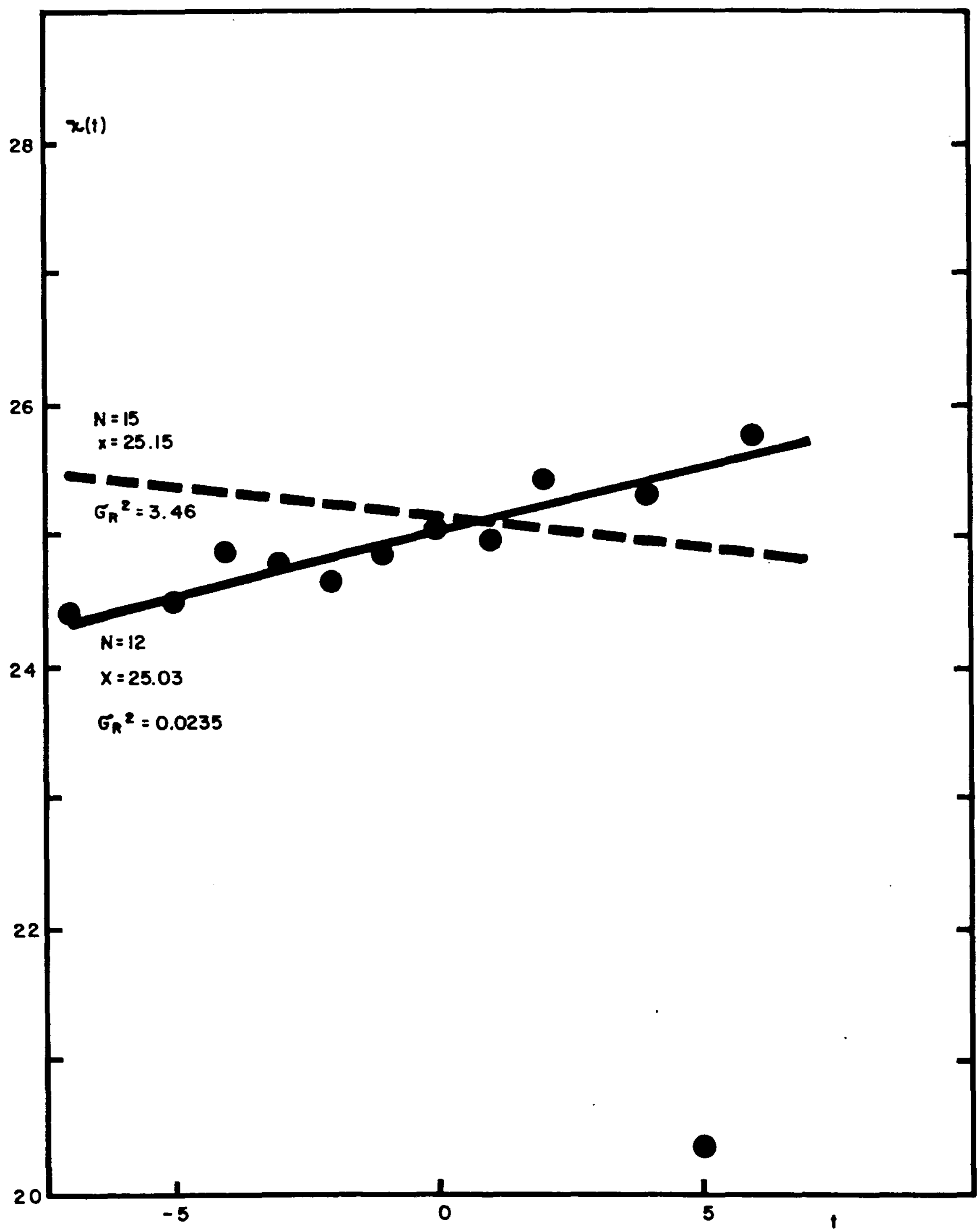

x(t)
28
26
24
22
20
N = 15
x = 25.15
σ_R^2 = 3.46
N = 12
X = 25.03
σ_R^2 = 0.0235
-5
0
5
t

FIGURE 4.1

(6) Increment n. If n=N+1, then n=1. Go to step (2).

The sequence of steps (1) through (6) continues until (5) is not executed in a complete pass through the data from n=l to n=N; in other words, until no more outliers are identified. The only parameter chosen by the user is "a predetermined threshold" in step (4). This threshold is a dimensionless quantity, the number of residual variances a datum may depart from the current least squares curve without being considered an outlier. 1f the underlying distribution of residuals is Gaussian, setting the threshold to 1 would initially tend to identify 31.8 percent of the data points as outliers; to 4, about 4.6 percent; and to 9, Ø.27 percent. However, these percentages would not be attained because of the increase in residual variance occurring as the denominator ($N-X-N_C$) decreases as in (4.5), the number of coefficients N_C. in that equation being 2. The data in Figure 4.1 were edited using this six step algorithm with i_n=1 for all n and the threshold set to 4 on input.

4.4.6.6 This procedure has accomplished the following:

- Outliers have been identified using a simple test which is not specific to any atmospheric variable.
- By shifting the origin of the times t_n so the series is time-centered on Ø, $a_Ø$ is the average value of the data excluding the outliers. rvar(x,z)/(N-M) is a statistically sound estimate of its uncertainty.
- a_1 is a statistically sound estimate of trend in the data. Its uncertainty may also be estimated.
- If it is necessary to replace the outliers with plausible values, the least squares fitted line readily provides them.

4.4.6.7 The procedure can be generalized in several respects:

- Least squares fit higher order polynomials or other functions.
- If one has other knowledge of the quality of a sample, such as obtains from signal-to-noise measurements, use this information to weight samples more realistically than "binary worth" discussed in 4.4.4.3.
- Code computationally efficient and stable matrix inversion algorithms.

In any case, the user has two choices to make - selecting a model for the data and the accept/reject threshold.

4.4.6.8 Choosing the order of the polynomial is equivalent to choosing a model which accurately fits the error and noise-free data. In general, one doesn't know the correct model, 50 one faces the consequences of a modelling error. Modelling error appears as an increase in residual variance. 1f the increase is small compared to the contribution of noice and other error sources in the data or small compared to the desired accuracy, then an adequate model has been selected. The following considerations help govern the choice of the model.

- Very long time series of atmospheric data are difficult to fit with polynomials of reasonable order. Break long series into shorter segments.
- Any N_C points can be exactly and meaninglessly fitted by a polynomial with N coefficients. 1f N is the number of points to fitted, choose $N_C < N/3$.

4.4.6.9 Selecting the number of residual variances as accept/reject threshold involves the following considerations:

- Too large a threshold will result in no discrimination against outliers.
- Too small a threshold will result in the excessive rejection of valid data.
- Let N_r be the number of residual variances chosen as the accept/reject criterion, N the number of data points to be fitted, and N_C. the number of coefficients in the polynomial. For any rejection to occur, $N_r<(N-N_C)$. $4<=N_r<=(N-N_C)/2$ seems to be a workable range.

4.4.6.1Ø The reader should also be aware that this approach to editing data suffers from a potentially serious flaw. The identification of erroneous samples depends upon the departure of samples from a curve fitted to erroneous data. In practice, most of the difficulty is avoided by requiring that the independent variable (usually time) be error-free and reasonably well distributed across the total range it spans. Equispaced sampling satisfies this requirement. An alternative approach to editing has been proposed by Fischler and Bolles (1981) to avoid this flaw. It appears to deal very effectively with data badly contaminated by large errors, but at the usual cost of increased computational complexity.

4.5 Inter-sensor Data Quality Control

4.5.1 Inter-sensor data quality control spans two scales - one within the confines of a single automatic weather station and a second spanning a network of stations. In either case, quality controls should be based upon established physical and meteorological principles.

4.5.2 In the interest of economy, an automatic weather station usually does not include redundant sensors and it tends, by design, to measure independent atmospheric variables. Thus, the list of inter-sensor controls within a station would likely be small. Some examples are:

- Dewpoint temperature cannot exceed ambient temperature.
- Precipitation without clouds overhead or just having passed overhead is very unlikely.
- Nonzero wind speed and zero wind direction variance strongly suggests a wind direction sensor problem. Conversely, zero average wind speed and nonzero wind direction variance suggests a defective wind speed sensor.

4.5.3 At the network level, some very powerful tests are available. Meteorological fields have considerable spacial continuity on the synoptic scale. These fields can be analyzed at a central facility to uncover anomalous observations and trigger investigative or remedial action. Because of the low level of turbulent fluctuations in pressure and the confidence with which local geographic influences can be removed by normalizing all observations to a common reference level, this atmospheric variable is a prime candidate for this type of data quality control. By time averaging over many observations, other variables should be susceptible to this analysis as well. However, local orographic effects must be carefully considered and taken into account.

5.Ø AVERAGES AND OTHER STATISTICS

5.1 An examination of WMO SYNOP and SHIP codes reveals that the following meteorological statistics are needed:

- Averages, or quantities derived from averages, of time series of visibility, wind direction, wind speed, temperature, dewpoint temperature, sea level pressure, pressure change, precipitation, and maximum and minimum temperatures.
- Statistics requiring much more sophisticated processing include cloud height, cloud cover, wind wave period and height, and swell direction and period and height.

Other elements of the synoptic observation are beyond the capability of automatic observation at this time. These are State of the sky, cloud layer direction of motion, state of the ground, ice or snow depth, cloud type, and special phenomena.

5.2 Averages

5.2.1 An earlier section established the need to linearize before averaging. Note that any linear transformation may precede or follow averaging without consequence. In other words, if y is a linear function of x:

(5.1) $y=a_{\emptyset}+a_1x$, where $a_{\emptyset}$ and a_1 are constants,

then the averages of y and of x, Y and X respectively, are related by:

(5.2) $Y=a_{\emptyset}+a_1X$.

5.2.2 Averaging using analog circuitry

5.2.2.1 Define the arithmetic average of a time dependent variable x(t'), X(t,T), as:

$$(5.3)\quad X(t,T)=(1/T)\int_{t-T}^{t} x(t')dt'$$

Define the exponential average of x(t'), $X_E(t,T_C)$, as:

$$(5.4)\quad X_E(t,T_C) = \int_{-\infty}^{t} x(t')\exp(t'/T_C)dt' / \int_{-\infty}^{t} \exp(t'/T_C)dt'$$

In (5.4), T_C is referred to as the time constant.

5.2.2.2 Why define two averages? Simply because the arithmetic average conforms to the normal meaning of average and is readily implemented digitally, while the exponential average is the simplest low-pass filter, represents the simplest response of a sensor to atmospheric fluctuations, and is much more convenient to implement in analog circuitry than the arithmetic average. Figure 5.1 shows the implementations of both averages. Acheson (1968) demonstrated that when T_C equals approximately Ø.5T in the range of 1<=T<=1Ø minutes, arithmetic and exponential averages of wind speed are essentially indistinguishable. The physical basis for this observation and subsequent experience suggests that the two averages are also indistinguishable for atmospheric pressure, temperature, humidity, extinction coefficient, and wind direction as well.

5.2.3 Averaging using digital logic

5.2.3.1 Given a time series of discretely sampled data x_j, x_{j+1}, x_{j+2},... sampled at times t_j, t_{j+1}, t_{j+2},...; then the arithmetic average of x is $X(t_k,T)$:

$$(5.5)\quad X(t_k,T)=(1/N)\sum_{j=k-N+1}^{k} x_j, \qquad \text{where } T=t_k-t_{k-N+1}.$$

Analog and digital arithmetic averages are identical if equispaced sampling is used ($t_{j+1}-t_j$=constant for all j) and the sampling rate exceeds the cutoff frequency of the data ($t_{j+1}-t_j < 1/f_C$). As noted earlier, satisfying these conditions also guarantees that the average will have minimum uncertainty.

5.2.3.2 While not often used, the exponential average can also be expressed digitally for equispaced sampled data.

$$(5.6)\quad X_E(t_k,r)=(1-r)x_k+rX_E(t_{k-1},r)$$

$$\text{where } r=T_C/(T_C+t_k-t_{k-1}).$$

5.2.4 Mixed analog and digital averaging

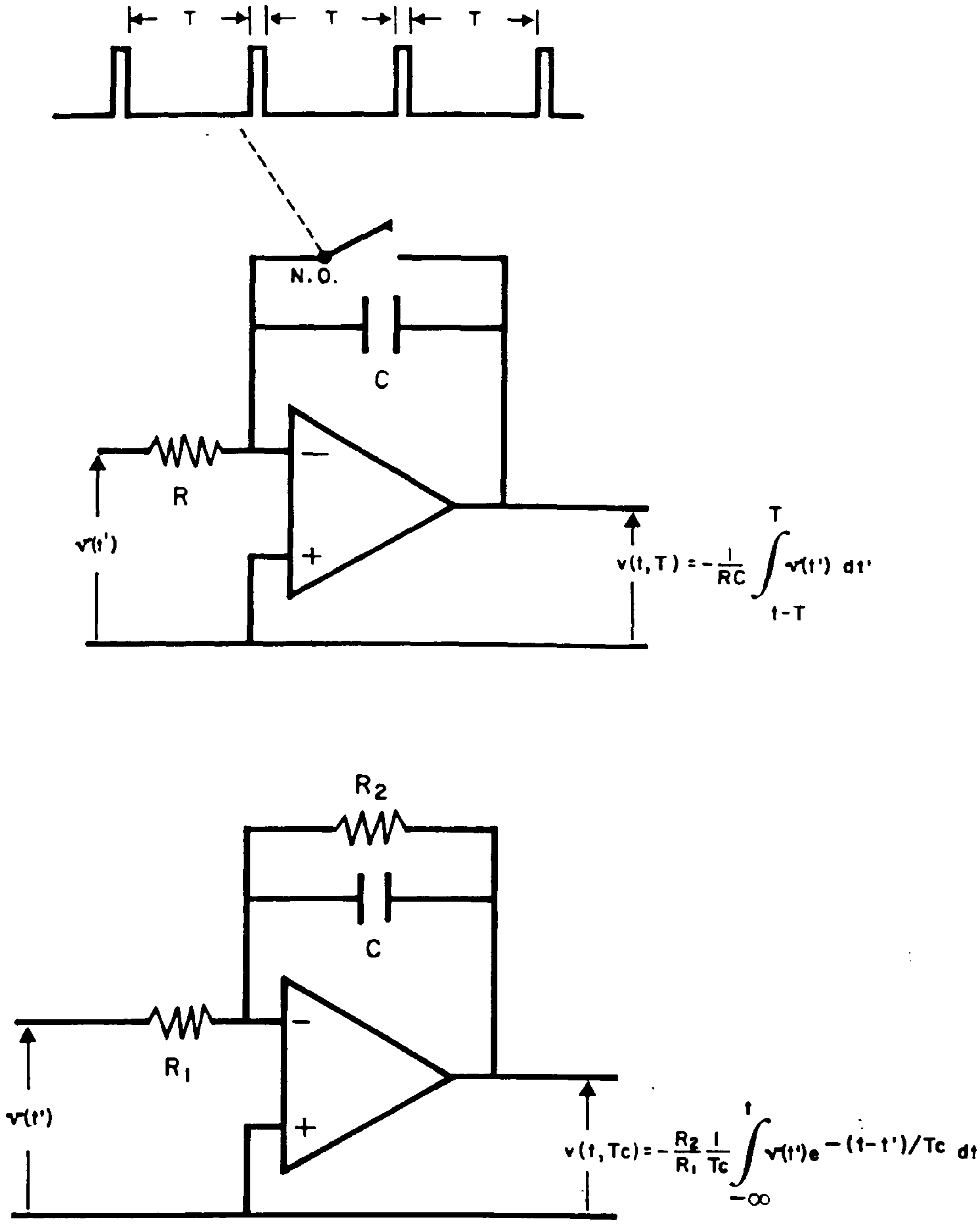
5.2.4.1 For very fast response sensors, transducer outputs vary rapidly and the information bandwidths are large, necessitating high sampling rates for optimal (minimal uncertainty) averaging. To reduce the sampling rate and still provide the optimal digital average, linearize the transducer output (if required) exponentially average it using analog circuitry with time constant T_C then sample at intervals T Also note that it desirable, and sometimes necessary, to low-pass filter analog-to digital converter inputs to minimize the effects of noise and line pickup. This is most simply done with an analog circuit forming an exponential average. The cutoff frequency at its output is approximately $1/2T_C$.

5.3 Cloud Height and Cloud Cover

5.3.1 To obtain statistically sound height and coverage estimates of cloud layers, a considerable number of measurements are required. Since ceilometers take an appreciable amount of time, typically 1Ø to 3Ø seconds, to make a single determination of cloud height, about 3Ø minutes are needed to acquire 5Ø to 1ØØ measurements. During an interval of this length, the detection of clouds in a higher layer is almost certain if a lower broken or scattered layer exists and the higher layer is within range of the ceilometer. Under these conditions, the resultant time series of cloud heights characterizes two or more layers. A simple average of all heights is clearly incorrect. A procedure for grouping heights prior to averaging heights within each group must be used. This grouping will also allow the estimation of cloud cover by layer by considering the ratios of the numbers of observations of clouds in each group to the total number of possible observations.

5.3.2 If one regards each cloud layer as a probability distribution with unknown mean and variance, both changing with time in the general case, then measurements of cloud base height are samples drawn from an unknown number of distributions with unknown parameters. Grouping cloud base heights equates to estimating the number of distributions and their means and variances. In general, this is a formidable problem. The subject of "cluster analysis" within the broader topic of "pattern recognition" probably offers the most well-documented source of possible solutions to this problem.

5.3.3 Duda et. al. (1971) studied the cloud base height estimation problem. One of their suggested approaches, hierarchical clustering, has formed the basis for a number of algorithms in limited use within the USA. These algorithms simplify the problem by assuming that:



$T_C = R_2C$

FIGURE 5.1

- 6Ø ceilometer measurements taken at 3Ø second intervals over a 3Ø minute period constitute a minimally acceptable sample,
- the mean cloud base heights of every layer are constant during the observing period,
- no more than 5 cloud layers exist within range of the ceilometer,
- and any group or cluster containing 5 or fewer observations does not describe a layer of clouds.

The following paragraphs describe the basis for the algorithm in plain language without detail and much more precisely in a pseudo-computer language. The input to the algorithm is a set of measured cloud heights h_j, j=1,2,...N.

5.3.4 In plain language:

Initially, each cluster contains one cloud height. (1> Find the distances between clusters.

(2) Merge the pair of clusters separated by the minimum distance to create a new cluster and delete the merged pair.
(3) Continue this process until only 5 clusters remain.
(4) Merge clusters which are very close together.
(5) Disregard any remaining clusters which contain 5 or fewer observations.

5.3.5 In pseudo-language (let H_j represent the average cloud base height and n_j the number of observations in the j^{th} cluster and all heights are in feet):

```
Initialize:   SORT the observations so h1<=h2<=...<=hN
           FOR j=1 to N
              Hj=hj
              nj=1
              NEXT j
           j1=1

(1) Dmin^2 = largest positive number available
    FOR j = j1 to N-1
        D^2 = nj nj+1(Hj+1-Hj) ^2/( nj+ nj+1)
        IF D^2 < Dmin^2 then Dmin^2 = D^2 and k=j
        NEXT j

(2) Hk = (nkHk+ nk+1Hk+1)/( nk+ nk+1)
    nk = nk+ nk+1
```

$H_{k+1} = 0$
$n_{k+1} = 0$

(3) SORT the clusters so $H_1 <= H_2 <= ... <= H_N$
$j_1 = j_1 +$
IF $N - j_1 > 4$ then go to (1)

(4) $n_m = 0$
FOR $j = j_1$ to $N-1$
IF $H_j < 1000$ and $H_{j+1} - H_j <= 250$
or $1000 <= H_j < 3000$ and $H_{j+1} - H_j <= 350$
or $3000 <= H_j < 5000$ and $H_{j+1} - H_j <= 450$
or $H_j >= 5000$ and $H_{j+1} - H_j = 600$
then $H_j = (n_j H_j + n_{j+1} H_{j+1}) / (n_j + n_{j+1})$
$n_j = n_j + n_{j+1}$
$H_{j+1} = 0$
$n_{j+1} = 0$
$n_m = n_m + 1$
NEXT j
IF $n_m = 0$ then go to (5)
$j_1 = j_1 + n_m$
SORT the clusters so $H_1 <= H_2 <= ... <= H_N$
GO TO (4)

(5) $n_d = 0$
FOR $j = j_1$ to N
IF $n_j <= 5$ then $H_j = 0$
NEXT j
SORT the clusters so $H_1 <= H_2 <= ... <= H_N$
$j_1 = j_1 + n_d$

At the conclusion of these steps, there are $N - j_1 + 1$ significant clusters, each describing one cloud layer.

6.Ø DERIVING RELATED VARIABLES AND THE ART OF APPROXIMATION

6.1 In preparing observations for coding and transmission, the observer consults conversion and other reference tables. It is often possible to incorporate tables directly into an automatic weather station and provide interpolation routines. Doing this exacts a penalty in the form of storage requirements and ensuring the accuracy of the tabular data and the interpolation routine. Sometimes simple formulae constitute the basis for the table. If this is recognized they may be used directly in station software. In other cases, formulae are not known, are too complex or time consuming in execution, or require the use of functions not available in the system in use. In any of the latter cases, one appeals to the art of approximation.

6.2 Finding appropriate formulae or good approximations does not require extraordinary mathematical skills - a knowledge of a few general references and a willingness to search literature almost always suffices. Two widely available references stand out - The Smithsonian Meteorological Tables for meteorological formulae and the Handbook of Mathematical Functions for approximations.

6.3 In the following subsections, some formulae and approximations for humidity, reduction of station to sea-level pressure, altimeter setting, and visibility are presented and discussed.

6.4 Humidity

6.4.1 In converting values of dewpoint temperature to or from any other measure of humidity, an expression relating temperature and saturation water vapor pressure is needed. No exact formula exists. The best equations are based upon integration of the Clausius-Clapeyron equation, a fundamental thermodynamic relationship. These results include the Goff-Gratch formulation, adopted by the IMO in 1947, and more recent work conducted at the National Bureau of Standards in the USA by Wexler and Greenspan (1971). The latter proposed equations of the form:

$$(6.1)\quad \ln(e_w) = \sum_{j=0}^{n} E_j T^{j-1} + B \ln(T)$$

where e_w=saturation water vapor pressure
T=absolute temperature
E_j,B=constants

For n=3, 4, or 5, this equation yields excellent results over the temperature range of Ø to 1ØØ ^{o}C and good agreement, within Ø.Ø4 Pascal or Ø.ØØØ4 millibars, with the Goff-Gratch formulation between Ø and -4Ø ^{o}C. For the case n=3 (e_w in Pascals and T in deqrees Kelvin):

$E_Ø$ = -6.77772Ø3x1Ø^{3}

E_1 = 5.44Ø9359xlØ^{1}

E_2 = -8.Ø4Ø4l43xlØ$^{-3}$

E_3 = 7.15445Ø3xlØ$^{-6}$

B = -3.8358214

6.4.2 Lowe (1977) gives an empirical polynomial expression which is simpler than (6.1):

(6.2) $e_w=a_Ø+t(a_1+t(a_2+t(a_3+t(a_4+t(a_5+a_6t)))))$

$a_Ø$ = 6.1Ø7799961

a_1 = 4.436518251x1Ø$^{-1}$

a_2 = 1.4289458Ø5x1Ø$^{-2}$

a_3 = 2.65Ø648471xlØ$^{-4}$

a_4 = 3.Ø3124Ø396xlØ$^{-6}$

a_5 = 2.Ø34Ø8Ø948X1Ø$^{-8}$

a_6 = 6.13682Ø929xlØ$^{-11}$

e_w is in millibars when t is in degrees Celcius

The agreement of (6.2) with the Goff-Gratch formulation is also excellent, within Ø.1 Pascal or Ø. ØØ1 millibar from -4Ø to +5Ø^{o}C.

6.4.3 Equations (6.1) and (6.2) are designed to compute vapor pressure given temperature. Either can be easily used to find temperature given vapor pressure by applying Newton's method of successive approximation. Using (6.1) as an example:

(6.3) $T_k= T_{k-1}-f(T_{k-1})/(df/dT)_{k-1}$

where k indexes the trial, k=1,2,3,...
$T_Ø$ is an arbitrary initial guess temperature

n

$$f(T) = \sum_{j=0} E_j T^{j-1} + B \ln(T) - \ln(e_w)$$

$$(df/dT)_{k-1} = \sum_{j=0}^{n} (j-1) E_j (T_{k-1})^{j-1} + B/T$$

Iteration continues until $T_k - T_{k-1}$ is acceptably small. As an example, for e_w=12338.49 Pa and T_0=273.15 K (0 ^{o}C).

k	T_k-273.15	$T_k - T_{k-1}$
1	41.364	41.364
2	49.738	8.373
3	50.000	0.262
4	50.000	0.0002

6.5 Sea level Pressure

6.5.1 The *Manual of Barometry* (1963) gives the following equation for the reduction of station pressure p to sea level pressure p_0:

(6.4) $p_0 = p \exp(HK \ln(10)/T_{mv})$

where H=station elevation (geopotential meters)

K ln(10)=6.1454x10^{-2} (^{o}R/gpm)

T_{mv}=mean virtual temperature of the column of air between the station and sea level (^{o}R)

Since the column of air in the definition of T_{mv} is an imaginary one for land stations, it needs definition:

(6.5) $T_{mv} = 459.7 + t_s + aH/2 + C_h(H) e_s + F(H, t_s)$

where t_s= average station temperature (^{o}F), formed as the mean of current temperature and that observed 12 hours earlier

a = 0.0117 (^{o}F/gpm)

$C_h(H)$= humidity correction factor, a function of station elevation alone (^{o}F/mb)

e_s= current observed water vapor pressure (mb)

$F(H,t_s)$= correction for plateau effect and local lapse rate anomalies (^{o}F)

Tables 7.4.1 through 7.4.8 in the reference give values of the correction factor $F(H,t_s)$ for a large number of North American stations. For any station below 305 gpm (1000 feet) in altitude, the correction is condensed into a single table (7.4.1) as a function of the mean temperature t_S defined above and the annual normal station temperature t_{Sn}.

6.5.2 As an alternative to storing this empirical table and interpolating as needed, the data has been least-squares fitted with the result:

$$(6.6)\quad F(H,t_S) = F(t_{Sn}, t_S) = (-1.0+0.438 t_{Sn}) + (-0.403 + 1.25\text{x}10^{-6} t_{Sn}) t_S^2.$$

Equation (6.6) fits the tabulated data with a root-mean-square error of 0.13 $^{\circ}$F and a maximum error of 0.44°F.

6.6 Altimeter Setting

6.6.1 The _Smithsonian Meteorological Tables_ (1963) define altimeter setting A as:

$$(6.7)\quad A=(p-0.01)(1+p_0{}^n\ a/T_0)(H/(p-0.01)^n)^{1/n}$$

where p= station pressure (inches mercury or inHg)
p_0= 29.921 (inHg)
a= 0.0065(K/m)
T_0= 288 (K)
n= 0.190284

While (6.7) can be evaluated directly with a floating point processor capable of executing the needed operations, the following approximation has proven useful in applications where very limited processing capabilities were available.

6.6.2 Let A'=A+0.01. Then (6.7) is approximated to within 0.005 inHg for station elevations up to 3000 meters by:

$$(6.8)\quad A'=p[1+(aH/T_0)\ (p_0/p)^n]^{1/n}.$$

Expanding A' in a Taylor series about the station elevation H and standard atmosphere pressure at H, p_H:

$$(6.9)\quad A'(p) = A'(p_H)+(dA'/dp)_{H,p_H}(p-p_H)+R_2 = p_0+ (p_0/p_H)^{1-n}(p-p_H)+R_2$$

where $R_2=(d^2A'/dp^2)_{H,p'}(p-p_H)^2/2$

$$abs(p'-p_H)<=abs(p-p_H)$$

The remainder R_2 is maximized when p' is minimized. The following table gives values of R_2 for pressures corresponding to 26 inHg at sea level.

H (m)	H (inHg)	p(=p') (inHg)	R_2 (inHg)
Ø	29.92	26.ØØ	Ø Ø
1ØØØ	26.54	23.Ø6	$-4.1x10^{-3}$
2ØØØ	23.47	2Ø.4Ø	$-8.0x10^{-3}$
3ØØØ	2Ø.7Ø	17.99	$-11.8x10^{-3}$

Thus, for station elevations up to 3ØØØ meters, altimeter setting may be approximated to within Ø.Ø2 inHg with:

(6.1Ø) $A=29.911+(29.921/p_H)^{0.809716}(p-p_H)$

The coefficient $(29.921/p_H)^{0.809716}$ is a constant for all stations of the same height and may be readily precomputed.

6.7 Visibility

6.7.1 As noted in the earlier section on linearization, visibility is neither linear nor instrumentally measurable. The linear measurable atmospheric property is extinction coefficient. To convert average extinction coefficient to average visibility, the following two equations are required:

(6.11) by day: $V=-\ln(C_t)/C_E$

(6.12) by night: $E_t=I\ exp(-C_E V)/V^2$

where	V=average visibility	(miie) 1
	C_E=average extinction coefficient	($mile^{-1}$)
	C_t=standard threshold of contrast	(Ø.Ø55)
	E_t=standard threshold of illuminance	(Ø.Ø84/V mi-cd)
	I=standard source intensity	(25 candela)

Equations (6.11) and (6.12) and the definitions of the variables are standard and widely accepted, but the units and values given above are not universal. While (6.11) serves the need as written, (6.12) cannot be explicitely solved for visibility. Newton's method of successive approximations provides the solution. Using the same notation as in equation (6.3):

(6.13) $V_k=V_{k-1}-f(V_{k-1})/(df/dV)_{k-1}$

where $f(V)=0.084V-25\exp(-C_EV)$

$df/dV=0.084+25C_E\exp(C_EV)$

For $V_0=1$ and $C_E=79.98$ mile^{-1}, Newton's method produces the following resuits:

k	Vk	V_k-V_{k-1}
1	Ø.ØØØØ	-1.ØØØØ
2	Ø.Ø125	Ø.Ø125
3	Ø.Ø25Ø	Ø.Ø125
4	Ø.Ø375	Ø.Ø125
5	Ø.Ø5ØØ	Ø.Ø125
6	Ø.Ø623	Ø.Ø124
7	Ø.Ø744	Ø.Ø124
8	Ø.Ø855	Ø.Ø111
9	Ø.Ø943	Ø.ØØ88
10	Ø.Ø99Ø	Ø.ØØ47
11	Ø.1ØØØ	Ø.ØØ1Ø
12	Ø.1ØØØ	Ø.ØØØØ4

Care was taken in (6.13) to choose the form of f(V) to avoid difficulties with intermediate values of $V<=0$.

REFERENCES

Acheson, D.T., "An approximation to arithmetic averaging for meteorological variables", J. of Applied Meteorology, V7, N4, August 1968, pp 548-553

Duda, R.O., R.L. Mancuso, and P.F. Paskert, "Analysis techniques for describing the state of the sky through automation", Stanford Research Institute, Menlo Park, CA 94Ø25, Report No. FAA-RD-71-52, July 1971

Fischier, M.A. and R.C. Bolles, "Random sample consensus: a paradigm for model fitting with applications to image analysis and automated cartography" Communications of the ACM, V24, N6, June 1981, pp 381-395

Handbook of Mathematical Functions, M. Abramowitz and I.A. Stegun eds., National Bureau of Standards Applied Mathematics Series 55, U.S. Government Printing Office, Washington. D.C., 1964, 1Ø46 pp

Lowe, P.R., "An approximating polynomial for the computation of saturation vapor pressure", J. of Appued Meteorology, V16, Nl, January 1977, pp lØØ-1Ø3

Manual of Barometry, V1 and V2, 1st ed., U.S. Government Printing Office,Washington, D.C., 1963

The Smithsonian Meteorological Tables, 6th rev. ed., R.J. List ed., Smithsonian Institution, Washington, D.C., 1951, 527 pp

Wexler, A. and L. Greenspan, "Vapor pressure for water in the range Ø to 1ØØ^{O}C", J. of Research of the National Bureau of Standards, V75A, N3, May-June 1971, pp 213-23Ø