# The Lower Atmosphere

## 3.1. MOIST AIR

### a. Some Parameter Definitions

For many practical purposes, the air of the lower atmosphere can be considered as a mixture of perfect gases; in the present context these may conveniently be assumed to be dry air of constant composition and water vapor. The water vapor content of the air can be expressed in terms of the mixing ratio, defined as the mass of water vapor per unit mass of dry air,

$$m = \rho_v/\rho_d \tag{3.1}$$

where $\rho_v$ is the density of the water vapor and $\rho_d$ the density of the air without the water vapor. The specific humidity is defined as the mass of water vapor per unit mass of moist air,

$$q = \rho_v/\rho \tag{3.2}$$

where $\rho = \rho_v + \rho_d$. The relative humidity is the ratio of the actual mixing ratio and the mixing ratio in water vapor saturated air at the same temperature and pressure,

$$r = m/m^*. \tag{3.3}$$

This is nearly equal to $(e/e^*)$, the ratio of the actual vapor pressure and the equilibrium vapor pressure at saturation.

According to Dalton's law, the total pressure in a mixture of perfect gases equals the sum of the partial pressures, and each of the component gases obeys its own equation of state. Thus, the density of the dry air component is

$$\rho_d = \frac{p - e}{R_d T} \tag{3.4}$$

where $p$ is the total pressure in the air, $e$ the partial pressure of the water vapor, $T$ the ('absolute') temperature, and $R_d$, which is given in Table 3.1, is the specific gas constant for dry air. Similarly, the density of water vapor is

$$\rho_v = \frac{0.622e}{R_d T} \tag{3.5}$$

where $0.622 = (18.016/28.966)$ is the ratio of the molecular weights of water and dry air.

The density of moist air is from (3.4) and (3.5)

TABLE 3.1
Some physical constants

| | |
|---|---|
| Dry Air | |
| Molecular weight: | 28.966 g mol$^{-1}$ |
| Gas constant: | $R_d$ = 287.04 J kg$^{-1}$ K$^{-1}$ |
| Specific heat: | $c_{pd}$ = 1005 J kg$^{-1}$ K$^{-1}$ |
| | $c_{vd}$ = 716 J kg$^{-1}$ K$^{-1}$ |
| Density: | $\rho$ = 1.2923 kg m$^{-3}$ |
| | ($p$ = 1013.25 mb, $T$ = 273.16 K) |
| Water Vapor | |
| Molecular weight: | 18.016 mol$^{-1}$ |
| Gas constant: | $R_w$ = 461.5 J kg$^{-1}$ K$^{-1}$ |
| Specific heat: | $c_{pw}$ = 1846 J kg$^{-1}$ K$^{-1}$ |
| | $c_{vw}$ = 1386 J kg$^{-1}$ K$^{-1}$ |

*Note*: The values listed in Tables 3.1, 3.4 through 3.6 are adapted from the Smithsonian Meteorological Tables (List, 1971), where the original references are cited.

$$\rho = \frac{p}{R_d T}\left(1 - \frac{0.378e}{p}\right) \tag{3.6}$$

showing that it is smaller than that of dry air at pressure $p$. This means that water vapor stratification plays a role in determining the stability of the atmosphere. The equation of state of moist air can be obtained by eliminating $e$ from (3.4) and (3.5)

$$p = \rho T R_d (1 + 0.61\, q). \tag{3.7}$$

This indicates that the mixture behaves as a perfect gas provided it has a specific gas constant $R_m = R_d(1 + 0.61\, q)$ that is a function of the water vapor content. Therefore, (3.7) is often expressed as

$$p = R_d \rho T_V \tag{3.8}$$

where $T_V$ is the virtual temperature defined by

$$T_V = (1 + 0.61\, q)T. \tag{3.9}$$

It is the temperature dry air should have in order to have the same density as moist air with given $q$, $T$ and $p$.

For convenient reference, some common units and conversion factors are listed in Tables 3.2 and 3.3.